

Implementation in 2015

Kaisei Junior High School Mathematics Entrance Examination

Time allowed: 60 minutes Total points: 85

Notes

- 1 Do not open the cover until the starting chime sounds.
- 2 When the starting chime sounds, first write your examinee number in the designated place on the cover of the question paper and answer sheet, then check the number of pages in the question paper before starting.
- 3 If there are any unclear printed parts, raise your hand silently.
- 4 When the ending chime sounds, put down your pencil immediately.
- 5 Both the question paper and the answer sheet will be collected, so place both on your desk with the front side up.

[A]

We consider a special operation, which we'll call the "angle brackets" operation, written as $\langle X \rangle$. Here's how it works for different types of numbers:

For integers, $\langle X \rangle$ is simply equal to X . For example, $\langle 3 \rangle = 3$, and $\langle 10 \rangle = 10$.

For fractions between 0 and 1, we first simplify the fraction as much as possible. If the simplified fraction is $\frac{Y}{Z}$, then $\langle X \rangle = \langle \frac{Y}{Z} \rangle = Y + Z$. For example, $\langle \frac{2}{3} \rangle = 2 + 3 = 5$, and $\langle \frac{8}{10} \rangle = \langle \frac{4}{5} \rangle = 4 + 5 = 9$.

For fractions greater than 1, we first convert them to mixed numbers and simplify the fractional part. If the result is $Y \frac{Z}{Y}$, then $\langle X \rangle = \langle Y \frac{Z}{Y} \rangle = Y + Z + W$. For example, $\langle 2 \frac{5}{6} \rangle = 2 + 5 + 6 = 13$, and $\langle \frac{42}{10} \rangle = \langle 4 \frac{1}{5} \rangle = 4 + 1 + 5 = 10$.

Now, answer the following questions based on these rules.

(1) Find the integer values of X that satisfy each equation:

$$(a) \langle \frac{35}{5} \rangle = X \quad (b) \langle X \frac{15}{12} \rangle = 30 \quad (c) \langle \frac{4}{X} \rangle = 12$$

(2) Identify all integers or fractions that satisfy $\langle X \rangle = 5$. List these values in ascending order, without repeating any value.

(3) Consider fractions with a denominator of 27 and numerators ranging from 1 to 2015 (i.e., $\frac{1}{27}, \frac{2}{27}, \frac{3}{27}, \dots, \frac{2014}{27}, \frac{2015}{27}$). From this set, we will select all fractions that satisfy $\langle X \rangle = 54$ and place these selected fractions in ascending order. Find the fifth number from the smallest and the fifth number from the largest, respectively.

[Answer]

[A] (1) (a) 12 (b) 24 (c) 44

(2) $\frac{1}{4}$, $\frac{2}{3}$, $1\frac{1}{3}$, $2\frac{1}{2}$, 5

(3) $\frac{209}{27}$, $\frac{1167}{27}$

[B] [1] 2 [2] 5 [3] 2 [4] d [5] 4 [6] 4 [7] 2 [8] 2

[C] (1) Kai: $6\frac{1}{7}$ minutes, Sei: $5\frac{20}{21}$ minutes

(2) $78\frac{4}{7}$ m (3) point B, $9\frac{11}{21}$ m (4) 19 times, 7.2 m

[D] (1) 24 cm^3 (2) 6 cm^2 , Refer to the figure.

(3) 12 cm^3 (4) 6 cm^3

